WADD TECHNICAL REPORT 61-25

CRITERIA FOR COMPARING THE EFFECTIVENESS OF DAMPING TREATMENTS

D. J. Mead

University of Southampton
Hampshire, England

JANUARY 1961

WRIGHT AIR DEVELOPMENT DIVISION

NOTICES

WADD TECHNICAL REPORT 61-25

CRITERIA FOR COMPARING THE EFFECTIVENESS OF DAMPING TREATMENTS

D. J. Mead

University of Southampton
Hampshire, England

JANUARY 1961

Materials Central
Contract No. AF 61(052)-332
Project No. 7351

WRIGHT AIR DEVELOPMENT DIVISION
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

600 – May 1961 – 29-1094

FOREWORD

This report was prepared by the University of Southampton, Department of Aeronautics and Astronautics, Southampton, U. K. under USAF Contract No. AF 61(052)-332. The contract was initiated under Project No. 7351, "Metallic Materials", Task No. 73521, "Behavior of Metals". The contract was administered by the European Office, Air Research and Development Command. The work was monitored by the Materials Central, Directorate of Advanced Systems Technology, Wright Air Development Division, under the direction of Mr. W. J. Trapp.

ABSTRACT

In this report, expressions are derived for the response of simple vibrating systems, from which criteria have been deducted to indicate the effectiveness of a damping treatment in attenuating the response. The criteria include factors by which the treatment increases the mass and stiffness of the system, together with the loss factor increment. The response quantities considered include bending stresses, accelerations, inertia forces and sound transmission associated with simple vibrating plates under harmonic and random excitation. Coincidence sound transmission is also briefly considered. It is shown that whereas the mass and loss factor increase is always advantageous, a stiffness increase in some instances is detrimental.

As an example, three different commercial treatments are compared on the basis of some of the criteria. With low treatment weights, the treatment providing the highest loss factor is superior judged by each criterion, but at higher weights according to some criteria a treatment having a lower stiffness, density and loss factor is more effective. The existence of optimum treatment weights for maximum effect upon the response is also shown by some criteria.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

W. J. Trapp

W. J. TRAPP
Chief, Strength and Dynamics Branch
Metals and Ceramics Laboratory
Materials Central

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF SYMBOLS

B	flexural stiffness per unit width of untreated plate
c	speed of sound in medium surrounding plate
$C_1, C_2, C_3,$	constants associated with the pressure distribution due to boundary layer turbulence
D	initial viscous damping coefficient
D_{crit}	critical viscous damping coefficient of untreated system
$F(t)$	random generalized force
i	$\sqrt{-1}$
k_ω, k_θ	symbols defined in text (equations III.8 and III.9)
L	length of side of square plate
K	initial generalized stiffness of system
m	mass per unit area of plate, before treatment
M	generalized mass of system
μm, μM	mass per unit area and generalized mass after treatment
P	amplitude of harmonic generalized force
p_i	amplitude of harmonic incident pressure
$\langle p^2 \rangle$	mean square random incident pressure
$\langle p_r^2 \rangle$	mean square random radiation pressure
p_t	amplitude of harmonic transmitted pressure
q, q', q'',	generalized displacement, velocity and acceleration
$\bar{q}$	amplitude of harmonic displacement
q_{rms}	root mean square value of random displacement
$\langle q^2 \rangle$	mean square value of random displacement
t	time
v	transverse bending displacement of plate
$\bar{v}$	amplitude of harmonic bending displacement
$w_f(\omega)$, $w_q(\omega)$	power spectral density of generalized force and generalized displacement respectively

$w_f(\omega_n)$	power spectral density of generalized force at the frequency ω_n.
x	lengthwise co-ordinate along plate
y	distance between neutral surface of plate and free surface before treatment
y	distance between neutral surface of plate and free surface after treatment
Z	complex mechanical impedance
α	factor by which treatment increases the distance between plate neutral surface and free surface
ϵ	phase angle
η_i	loss factor of system before treatment $(=2.D/D_{crit})$
θ	inclination of plane wave-fronts to plate surface
λ	wavelength of incident sound waves
λ_t	trace wavelength of incident sound waves on plate
μ	factor by which treatment increases the mass of the system
ρ	density of medium surrounding plate
$\sigma B(1 + i\eta)$	complex flexural stiffness of plate after treatment (per unit width)
$\sigma K(1 + i\eta)$	complex generalized stiffness of system after treatment
ω	circular frequency
ω_n	frequency of displacement resonance
ω_c	frequency at which coincidence transmission occurs

INTRODUCTION

The development of vibration damping treatments in recent years has led to treatments which greatly increase not only the damping of the structure to which they are applied, but also the overall stiffness of the structure. Formerly, the materials used were relatively soft and the stiffness increase was neglible. The amount by which the treatment increased the "damping ratio " or "loss factor" of the system was then a sufficient criterion by which to compare the effects of different treatments upon the response of the structure to the exciting forces. The treatment giving the greatest increase in the damping ratio would then also give the most attenuation of response, and the efficiency of the treatment could be judged by the "loss factor increment per given weight of treatment."

More recently, aircraft manufacturers have been considering the use of damping treatments as a means of attenuating acoustically excited vibration and consequent sound transmission and structural fatigue damage. Here, the basic structures to which the treatments are added consist of thin aluminium skins and sections, of which the Young's Modulus of elasticity is about one third of that of steel. The damping treatments give very large damping ratio increments together with very appreciable stiffness increases. Furthermore, the mass of the treatment may no longer be considered as negligible compared with that of the structure. Now the amplitude of the displacement of the structure under external harmonic excitation at resonance, is dependent not only upon the damping ratio of the system but also upon the stiffness. The amplitude of some of the other important response quantities (e.g. acceleration, velocity, etc.) are dependent in different ways upon the damping ratio , the mass and the stiffness of the system. When the excitation is of a random character (e.g. pressure fluctuations on an aeroplane structure due to a jet-efflux), these response quantities depend in yet

---

Manuscript released by the author December 1960 for publication as a WADD Technical Report.

further different ways upon the system characteristics. It follows, therefore, that the damping treatment will affect the response in several different ways by virtue of the stiffness and mass increase, as well as by virtue of the damping increase. Since the efficiency of a damping treatment must ultimately be judged by the effect the treatment has upon the response, it is evident that the damping ratio increment alone is an insufficient criterion by which to judge.

A theoretical investigation has already been carried out (ref.1) into the effect of a commercial damping treatment on structural vibrations (and the associated response quantities) excited by a jet-efflux. Allowance was made for the contributions of the treatment to the mass, stiffness and damping of the structure, but particular structural configurations were considered with particular initial conditions (e.g. the damping of the structure before treatment.)

This paper seeks in a more general way to derive criteria which provide a valid basis for assessing and comparing the effects of different treatments on a wider variety of response quantities than considered hitherto. Expressions are therefore derived (or quoted) for various response quantities of simple linear systems in terms of the mass, stiffness and damping of the systems. Both random and harmonic excitations of the systems are considered. From the expressions for the response, the combined effect of the mass, damping and stiffness of a damping treatment is readily determined and a corresponding criterion for comparing the effectiveness of different treatments may be deduced. For the systems under harmonic excitation the expressions derived relate to resonant conditions, and a comparison is sought between their magnitudes before and after the damping treatment is added. It must be recognised that, in general, the damping treatment will change the resonant frequency of the system. If the frequency of the harmonic exciting source does not change, then a system initially at resonance will be "de-tuned" and the resultant attenuation of the response will not necessarily be

due to the damping of the treatment. However, in most systems to which damping treatments are likely to be applied the frequency of excitation changes with changes of operating conditions, and there is bound to be some operating condition at which resonance of the treated system will occur. The response at this new condition should therefore be compared with the response at the untreated resonant condition, under the assumption that the amplitude of the exciting force is the same at both frequencies. This is, in effect, what is done in section II of this paper, where consideration is given to the amplitudes of harmonic displacement, velocity, acceleration, inertia force and the surface bending stresses of a vibrating plate. In all cases, the response in one mode of vibration only is considered, it being further assumed that the damping treatment does not change the mode.

Section III considers damping treatments in relation to harmonic sound transmission through simple structures. Once again, resonant conditions are assumed to pertain both before and after the treatment is applied, it being assumed that changing the operating conditions can always restore the system to a resonant state. Coincidence transmission through plates of infinite length is also considered, the change in the operating conditions required to restore coincidence after the treatment has been added being explained in the text.

In section IV random excitation of the system is considered and further criteria relating to displacement, acceleration, inertia force and bending stress are investigated. Here there is no need to consider changing operating conditions when the treatment is added to the system. The only assumption that is necessary is that the power spectral density of the exciting force is the same at the resonant frequencies of both the untreated and treated systems, and, furthermore, does not vary appreciably in the region of the resonant frequencies. This section also deals with the effect of a damping treatment upon the sound pressure transmitted

through two simple plate structures subjected to random pressure fluctuations.

Finally, in section V, a comparison is made between the effectiveness of three different commercial damping treatments, on the basis of the criteria deduced in the previous sections. The comparison is made assuming that equal weights of the treatments are used on a simple vibrating plate.

II CRITERIA APPLICABLE TO HARMONIC VIBRATIONS

II.A. Characteristics of the Mechanical System

Here we consider the effect of a damping treatment on the response of a system vibrating in a single natural mode of vibration under the action of a harmonic exciting force. Before the damping treatment is added, the generalized mass and stiffness of the system are M and K respectively. Suppose also that there exists a viscous damping mechanism, giving a generalized damping coefficient D. Denoting the exciting force by $P.e^{i\omega t}$ the equation of motion of the system (in terms of the generalized displacement, q) is

$$M\ddot{q} + D\dot{q} + Kq = P.e^{i\omega t}. \qquad \text{...II.1}$$

The damping treatment increases the mass and stiffness coefficients to $M\mu$ and $K\sigma(1+i\eta)$ respectively, the imaginary part of the complex stiffness being contributed by the "hysteretic" damping action of the treatment. If the vibrating system is a uniform flat plate attached to a rigid structure, and the damping treatment is in the form of a uniform layer over the plate, then μ is the factor by which the treatment increases the mass per unit area, and σ is the factor by which it increases the flexural stiffness of the plate. The equation of motion after treatment is therefore

$$M\mu\ddot{q} + D\dot{q} + K\sigma(1+i\eta)q = P.e^{i\omega t}. \qquad \text{...II.2}$$

II.B. The effect of the treatment on the resonant displacement amplitude.

Provided the initial damping (represented by D) is small compared with the added damping, the maximum amplitude of q (denoted by $\bar{q}_{max}$) occurs at the frequency

$$\omega_n = (K\sigma/M\mu)^{1/2}. \qquad \text{...II.3}$$

Then $$\bar{q}_{max} = P/(D\omega_n + K\sigma\eta) \doteq P/K\sigma\eta . \qquad \text{...II.4}$$

Now the maximum displacement of the system in its untreated condition can be expressed in the form $P/K\eta_i$ where η_i is twice the damping ratio (D/D_{crit}) corresponding to the initial damping. The effect of the damping treatment has therefore been to divide the initial resonant displacement amplitude by $\sigma\eta/\eta_i$. The effectiveness, and hence the efficiency, of a treatment used to attenuate vibration displacements is evidently measured by the value of the product $\sigma\eta$. In general, this product increases monotonically with the quantity of treatment added, whereas η approaches an asymptotic value. $K\sigma\eta$ is of course a true measure of the damping added to the system, whereas η is a function of the damping and stiffness. The criterion by which different treatments should be judged when used to attenuate harmonic vibration displacements is therefore the value of $\sigma\eta$ per given weight of treatment. Similarly, when the effect of increasing the amount of a given treatment is considered, the values of $\sigma\eta$ for the different amounts should be compared. The value of η itself is an insufficient criterion.

II.C. The effect of the treatment on the surface bending stresses of a vibrating plate.

To first order, the effect of the treatment on the displacement amplitude is the same as that on the amplitude of the stress within the system. However, the bending stress at the surface of a treated vibrating plate is proportional

to the product of the amplitude of curvature and the distance from the plate surface to the effective neutral surface of the section. The curvature is directly proportional to the displacement and therefore varies in inverse proportion to $\sigma\eta$.

Suppose the distance between the free plate surface and the neutral surface is initially y, and after treatment is αy. Since the amplitude of the oscillating curvature is proportional to the displacement amplitude, the surface bending stress of the system in its initial state is proportional to $yP/K\eta_i$ and in its final state to $\alpha y P/K\sigma\eta$. The final stress is therefore equal to the initial stress divided by $\sigma\eta\alpha^{-1}/\eta_i$. The factor $\sigma\eta\alpha^{-1}$ therefore represents the effect of the damping treatment, the larger it is the smaller being the bending stress. $\sigma\eta\alpha^{-1}$ is then the criterion by which different treatments should be judged when considering their effects upon surface bending stresses due to harmonic vibration at resonance.

Since the distance between the free surface and the plate neutral surface increases as the treatment thickness is increased, it obviously implies that the stresses do not decrease as rapidly as the displacement.

II.D. <u>The effect on the resonant velocity amplitude</u>.

The amplitude of the generalized velocity of the system is given by $\omega\bar{q}$ where $\bar{q}$ is the displacement amplitude at the frequency ω. This may easily be shown to have a maximum value at the frequency $\omega_n(1+\eta^2)^{1/4}$ when the assumption is made that the viscous damping is small compared with the hysteretic damping. At this frequency the velocity amplitude is given by

$$|\dot{q}|_{max} = P/K^{1/2}M^{1/2}\sigma^{1/2}\mu^{1/2}[2(1+\eta^2)^{1/2}-2]^{1/2}, \qquad \text{...II.5}$$

which, for small values of η reduces to

$$|\dot{q}|_{max} = P/K^{1/2}M^{1/2}\sigma^{1/2}\mu^{1/2}. \qquad \text{...II.6}$$

The maximum attenuation of harmonic velocity amplitude is obviously obtained when the product $\sigma^{1/2}\mu^{1/2}\eta$ is as large as possible. The mass of the treatment (included in the factor μ) is now important, but when comparing the effectiveness of equal weights of different treatments the significant parameter is the product $\sigma^{1/2}\eta$. The criterion by which different treatments should be judged when used to attenuate vibration velocity amplitudes is therefore the value of $\sigma^{1/2}\eta$ per given weight of treatment. When judging the effect of different quantities of the same treatment the complete expression $\sigma^{1/2}\mu^{1/2}\eta$ must be used as the criterion.

II.E. The effect on the resonant acceleration and inertia force amplitudes.

Acceleration amplitudes are of importance in connection with the effect on, say, items of electronic equipment mounted on a vibrating structure. Malfunctioning and failure of such equipment is caused by prolonged exposure to acceleration amplitudes above a certain level. Inertia force amplitudes are of importance in connection with acoustically excited aeroplane structures. Large harmonic inertia forces are reacted at the boundaries of the side panels of a fuselage in the vicinity of a propellor, and these may cause fatigue failures of rivets attaching the panels to the reinforcing structure. The effect of a damping treatment upon these quantities is therefore considered in this section.

The generalized acceleration amplitude under harmonic excitation is given by $\omega^2.\bar{q}$. This has a maximum value at the frequency $\omega_n\,(1+\eta^2)^{1/2}$ making the same assumption as before with regard to the magnitude of the viscous damping. At this frequency the acceleration amplitude is given by

$$|\ddot{q}|_{max} = P/M\mu\eta\,(1+\eta^2)^{-1/2} \qquad \text{...II.7}$$

reducing to

$$|\ddot{q}|_{max} = P/M\mu\eta \qquad \text{...II.8}$$

for small values of η . This is minimized by making $\mu\eta$ as large as possible. Considering equal weights of different treatments, the significant parameter is evidently η itself. Its value is therefore a sufficient criterion by which to judge the efficiency of a given quantity of treatment used to attenuate acceleration amplitudes.

The inertia force amplitude is directly proportional to the product of the generalized mass and the generalized acceleration amplitude. Its maximum value therefore occurs at the same frequency as the maximum acceleration, and is proportional to

$$P/\eta\,(1+\eta^2)^{-1/2} \qquad \text{...II.9}$$

reducing to P/η for small η . Again, the value of η for a given weight of treatment is a sufficient criterion by which to judge the efficiency when attenuating inertia force amplitudes.

It should be noted that as η becomes large, the term $\eta\,(1+\eta^2)^{-1/2}$ approaches unity. Any attempt to increase the value of η provided by a given treatment when η is already large, is not then accompanied by a worth-while reduction of acceleration or inertia force amplitude. It is probable, however, that when η is large enough for this effect to be important, the problems arising from the accelerations and inertia forces will have already been solved. The criteria developed in the whole of section I are summarized in table I.

III <u>CRITERIA APPLICABLE TO HARMONIC SOUND TRANSMISSION THROUGH SIMPLE STRUCTURES.</u>

As the addition of damping to a system has little effect upon forced vibrations apart from those occurring at resonance, this section will deal only with sound transmitted under structural resonant conditions and "coincidence" conditions. Two special cases only will be considered but these will serve to show the

different ways in which the mass, stiffness and damping properties of the treatment affect the transmitted sound pressure. This implies, of course, that different efficiency criteria are required for judging the merits of different treatments, depending on the nature of the transmission.

Two very simple transmission mechanisms will be considered:

(a) The sound transmitted through a finite flexible plate set in an otherwise rigid and infinite wall (or baffle). An incident field of plane harmonic sound waves impinges on one side of the plate causing resonance in one of its natural modes. The sound wavelength is assumed to be large compared with the plate dimensions.

(b) The sound transmitted through an infinite flexible plate when an infinite field of plane harmonic waves impinges on one side, causing "coincidence" transmission to exist.

III.A The sound transmitted through a finite plate.

As stated above, the plate is considered to be mounted in an infinite rigid wall. Firstly it is assumed that free field conditions exist on both sides of the wall, and on one side the incident field exerts an oscillating pressure on the plate. It is further assumed that the wavelength of the sound radiated by the motion of the plate is large compared with the plate dimensions. (This is inevitable if the incident sound wavelength is large, as already assumed). Insofar as the transmitted sound pressure in the far field is concerned, the oscillating plate may now be regarded as a simple source having a strength given by twice the product of the plate area and the average plate velocity amplitude. Now the sound pressure at a large distance from a simple source is proportional to the product of the source strength and the frequency, i.e. to ω times the plate generalized velocity amplitude, which is the same as the amplitude of the generalized

acceleration of the plate. The latter is given by equations II.7 and II.8 in which P is now to be interpreted as the generalized force corresponding to the incident sound pressure, and $M\mu$ as the generalized mass of the plate corresponding to the given mode. Then the maximum transmitted pressure is proportional to

$$P/M\mu\eta\,(1+\eta^2)^{-1/2} \quad .$$

This assumes that the acoustic radiation damping is small compared with the damping arising from the treatment, an assumption which is justifiable for any conceivable plate. It is evident therefore that for a given weight of damping treatment, the transmitted pressure is inversely proportional to $\eta(1+\eta^2)^{-1/2}$ The value of η is then a sufficient criterion by which to judge the effectiveness of equal weights of different treatments.

Now if one side of the plate is enclosed by a reverberant chamber, the natural modes of the plate will couple with standing waves within the chamber. There are, in fact, an infinite number of standing waves with which any one plate mode may couple, implying that there is an infinite set of natural frequencies at which the mode may resonate. (See, for example, refs. 2,3). It is required to establish, therefore, the relationship between the resonant pressure amplitude within the chamber, corresponding to any one of the standing wave systems, and the generalized plate characteristics (including μ, σ and η). A preliminary investigation has been carried out by the author (the work to be published later) considering a rectangular chamber, one wall of which consisits of the flexible plate assumed to have simply supported edges. The other walls were considered to be rigid. The results of the analysis suggest the following effects of increasing μ, σ and η :

The sound pressure at each of the standing wave resonances is inversely

proportional to η . The effect of increasing μ and σ may be combined by considering the effect they have on the uncoupled natural frequency of the plate, and then examining the effect of change of frequency. If the coupled standing wave - plate natural frequency is much less than that of the uncoupled plate, then an increase of the plate frequency tends to increase the resonant sound pressure. If, on the other hand, the coupled frequency is considerably greater than the plate frequency, then increasing the plate frequency decreases the resonant sound pressure. When the coupled frequency is close to the plate frequency, no such generalisation may be made and each case must be considered on its own merits.

III.B. The sound transmitted by coincidence through an infinite plate.

We now consider the effect of a damping treatment on the sound pressure transmitted through an infinite plate (or beam), on one side of which is an incident sound field of plane harmonic waves whose wave-fronts are inclined at an angle θ to the plate surface. Before the treatment is added, coincidence transmission exists, the trace velocity of the incident field coinciding exactly with the phase velocity of the flexural wave in the plate excited by the incident field. The transmitted pressure is then equal to the incident sound pressure. The mass and stiffness of the damping treatment changes the phase velocity of the flexural wave, and if the incident sound field remains the same, a de-tuning effect will reduce the transmitted pressure. The reduction due to the de-tuning may be considerably greater than that due to the additional damping. If, however, the

inclination or the frequency of the incident field is changed (the incident pressure remaining constant) and coincidence conditions are restored, then a measure of the effectiveness of the damping treatment may be found by comparing the transmitted sound pressure under these new coincidence conditions with the pressure under the initial conditions.

In the subsequent analysis, a unit width of the treated plate need only be considered. This has a complex flexural stiffness $\sigma B(1 + i\eta)$ and a mass per unit length μm. The imaginary part of the complex stiffness represents the internal damping due to the damping treatment. B and M are the flexural stiffness and mass per unit length before the treatment is added. σ & μ are the stiffness and mass factors representing the effect of the treatment.

Let the incident pressure amplitude be p_i. Due to reflection of the incident wave and consequent pressure doubling, the incident pressure acting on the plate is given by

$$2p_i . \exp i(\omega t + 2\pi x/\lambda_t) . \qquad \text{...III.1}$$

(See figure 1 for explanation of undefined symbols). If it is assumed already that the flexural wave in the plate is of harmonic form, and of wavelength λ_t, it may be shown that the transmitted (or re-radiated) pressure, p_t, acting on the plate surface is

$$\rho c . \cos\theta . \partial v/\partial t \qquad \text{...III.2}$$

where v is the local transverse bending displacement of the plate. As this re-radiation occurs from each side of the plate, the net re-radiation pressure acting on the plate is $2\rho c . \cos\theta . \partial v/\partial t$.

The equation for the forced motion of the strip may now be written

$$\sigma B(1+i\eta).\partial^4 v/\partial x^4 + \mu m.\partial^2 v/\partial x^2 = -2p_i.\exp i(\omega t + 2\pi x/\lambda_t) - 2\,\partial v/\partial t\,\rho c.\cos\theta \qquad \ldots\text{III.3}$$

Putting $v = \bar{v}\exp i(\omega t + 2\pi x/\lambda_t + \epsilon)$, equating the real and imaginary parts on both sides and eliminating ϵ, it is found that

$$\bar{v} = 2p_i\left[\{\sigma B(2\pi/\lambda_t)^4 - \mu m\omega^2\}^2 + \{\sigma B(2\pi/\lambda_t)^4\eta + 2\rho c.\omega\cos\theta\}^2\right]^{-\frac{1}{2}}. \qquad \ldots\text{III.4}$$

For small η, it is sufficiently accurate to consider coincidence transmission occurring at the frequency ω_c, given by

$$\sigma B(2\pi/\lambda_t)^4 - \mu m\omega_c^2 = 0 . \qquad \ldots\text{III.5}$$

The local plate velocity and the incident sound pressure are then exactly in phase and the transmitted sound pressure is close to its maximum value, which actually occurs at a slightly greater frequency.

At the frequency ω_c, the transmitted sound pressure amplitude is $\omega_c\bar{v}\rho c.\cos\theta$ Substitution for $\bar{v}$ and rearrangement yields

$$p_t = p_i\left[1 + \frac{\sigma B(2\pi/\lambda_t)^4\eta}{2\rho c\,\omega_c\cos\theta}\right]^{-1}. \qquad \ldots\text{III.6}$$

Now the trace wavelength λ_t is related to ω_c by

$$\left.\begin{aligned} \lambda_t\sin\theta &= \lambda = 2\pi c/\omega_c \\ &\text{and also by} \\ \omega_c^2 &= (\sigma B/\mu m)(2\pi/\lambda_t)^4. \quad \text{(from III.5.)} \end{aligned}\right\}\ldots\text{III.7}$$

Using these relationships, λ_t and ω_c, or λ_t and θ may be eliminated from equation III 6. We then have

$$p_t = p_i\left[1 + \frac{c^2.\operatorname{cosec}^2\theta}{2\rho c.\cos\theta}.\eta(\mu m)^{3/2}(\sigma B)^{-\frac{1}{2}}\right]^{-1} = p_i/k_\theta \qquad \ldots\text{III.8}$$

or

$$p_t = p_i\left[1 + \frac{\omega_c}{2\rho c}\left\{1 - (c^2/\omega_c)(\mu m/\sigma B)^{1/2}\right\}^{-1/2} \cdot \mu m \eta\right]^{-1} = p_i / k_\omega \qquad \text{....III.9}$$

Equation III.8 may be used to indicate the effect of η, μ and σ on the transmitted pressure when the inclination of the incident field is kept constant while the frequency is varied to restore coincidence. Equation III.9 may be used to indicate the effect when the frequency is kept constant and the inclination is changed. It is implied here that coincidence transmission may indeed be restored. This may not always be possible, as suggested, for example, by the term $\left\{1 - (c^2/\omega_c)(\mu m/\sigma B)^{1/2}\right\}^{-1/2}$ in equation III.9. If μ/σ is such as to make this term vanish, or to be imaginary, coincidence transmission cannot occur at the particular frequency ω_c.

Now when η is zero (no internal damping), $p_t = p_i$. The pressure amplitude transmitted under damped conditions is therefore $1/k_\theta$ or $1/k_\omega$ times that transmitted under undamped conditions. The significant factors involved in k_θ and k_ω which must be made as great as possible in order to produce the greatest attenuation of pressure are:

$$\eta \mu \sigma^{-1/2} \qquad \text{...III.10}$$

and

$$\eta \mu \left[1 - (c^2/\omega_c)(m/B)^{1/2}(\mu/\sigma)^{1/2}\right]^{-1/2} \qquad \text{...III.11}$$

It is evident that η should be as great as possible for the greatest attenuation. Furthermore, both expressions indicate that it is desirable for σ to be as small as possible. An increase of stiffness counteracts, in some measure, the effect of the increased damping.

It may be noted that the term $(\sigma B/\mu m)^{1/2}$ is proportional to the resonant

frequency of a treated finite plate, when vibrating in a flexural mode of wavelength λ_t. This frequency is usually determined in an assessment test of the treatment. Each of the expressions III.10 and III.11 are reduced when this term increases, indicating that it is undesirable for the damping treatment to increase the resonant frequency of the system if transmitted sound pressure is to be minimized. If two different damping treatments provide equal loss factors, η, the most effective treatment will be that giving the lowest frequency increase (or highest decrease).

IV. CRITERIA APPLICABLE TO RANDOM VIBRATIONS.

IV.A. Random Mechanical Response Quantities

Once again, the response of a single degree of freedom system only will be considered. The system is excited by a force which varies randomly with time, and it is assumed that the power spectrum of the corresponding generalised force does not vary appreciably in the region of the natural frequency of the system. The equation of motion of the system, in terms of the generalised displacement, q, is

$$M\mu\ddot{q} + D\dot{q} + K\sigma(1 + i\eta)q = F(t) \qquad \text{...IV.1}$$

where F(t) represents the random exciting force. If the power spectral density of F(t) is denoted by $w_f(\omega)$, then it is well known that the power spectral density of the generalised displacement, $w_q(\omega)$, is given by

$$w_q(\omega) = w_f(\omega)/|Z|^2 \qquad \text{...IV.2}$$

$|Z|^2$ is the square of the modulus of the mechanical impedance,

i.e. $(K\sigma - \omega^2 M\mu)^2 + (\omega D + K\sigma\eta)^2$. ...IV.3

The mean square value of the generalised displacement, $\langle q^2 \rangle$ is given by $\int_0^\infty w_q(\omega)\, d\omega$.

Under the restriction quoted above upon the variation of $w_f(\omega)$, and assuming small total damping, the integral yields

$$\langle q^2 \rangle = w_f(\omega_n).(\pi/2)/K\sigma(D + K\sigma\eta/\omega_n) , \qquad \text{...IV.4}$$

where $$\omega_n^2 = K\sigma/M\mu . \qquad \text{...IV.5}$$

Assuming again that the initial viscous damping is very much smaller than the added hysteretic damping, we may write for the root mean square displacement

$$q_{rms} = \left[w_f(\omega_n)(\pi/2)\right]^{\frac{1}{2}} . \left[M^{\frac{1}{4}} K^{\frac{3}{4}} \mu^{\frac{1}{4}} \sigma^{\frac{3}{4}} \eta^{\frac{1}{2}}\right]^{-1} . \qquad \text{...IV.6}$$

Thus the r.m.s. displacement is inversely proportional to $\sigma^{\frac{3}{4}}\mu^{\frac{1}{4}}\eta^{\frac{1}{2}}$, and the corresponding efficiency criterion for a damping treatment is the value of $\sigma^{\frac{3}{4}}\eta^{\frac{1}{2}}$ per given weight of treatment. It follows, by direct comparison with section II.C, that the criterion relating to the surface bending stress of a treated plate is $\sigma^{\frac{3}{4}}\eta^{\frac{1}{2}}\alpha^{-1}$ per given weight of treatment.

Under the same assumptions, the root mean square value of the generalised random velocity is given by $\omega_n q_{rms}$ i.e.

$$\dot{q}_{rms} = \omega_n . q_{rms} = \left[w_f(\omega_n)(\pi/2)\right]^{\frac{1}{2}} . \left[M^{\frac{3}{4}} K^{\frac{1}{4}} \mu^{\frac{3}{4}} \sigma^{\frac{1}{4}} \eta^{\frac{1}{2}}\right]^{-1} . \qquad \text{...IV.7}$$

The r.m.s. velocity is therefore inversely proportional to $\sigma^{\frac{1}{4}}\mu^{\frac{3}{4}}\eta^{\frac{1}{2}}$, and the corresponding efficiency criterion is the value of $\sigma^{\frac{1}{4}}\eta^{\frac{1}{2}}$ per given weight of treatment.

Consider now the mean square value of the random generalised acceleration.

The power spectral density of the acceleration is given by ω^4 times the power spectral density of the generalised displacement. Integrating this from $\omega = 0$ to $\omega = \infty$, and making the same restrictions as above upon the variation of the power spectrum in the region of the natural frequency, the mean square value of the generalised acceleration is found to be

$$\langle \ddot{q}^2 \rangle = W_f(\omega_n)(\pi/2)(1-\eta^2)\left[M^{5/2} K^{-1/2} \mu^{5/2} \sigma^{-1/2} \eta\right]^{-1} + \langle F^2(t)\rangle / M^2 \mu^2 . \quad \ldots \text{IV.8}$$

The first of the two components of this expression may be said to derive from the resonant response of the system, and is therefore dependent upon the damping. The second component is associated with the inertia reaction of the system, depending only upon the mass of the system. The relative magnitudes of the two components are obviously critically dependent upon the shape of the spectrum of the generalised force. If the resonant component is small compared with the other, the effect of the damping treatment will be mainly that of its mass, i.e. mass law attenuation will pertain. If the resonant component forms an appreciable part of the total response ,then the criterion $\sigma^{-1/2}\eta/(1-\eta^2)$ may be used to compare the effects on random acceleration of equal weights of different treatments. It is convenient to take the square root of this criterion and since η is usually very much less than one, it becomes $\sigma^{-1/4}\eta^{1/2}$. The force exerted by a randomly excited system on its supports consists of the sum of the exciting forces and the inertia forces corresponding to the response of the system. If the system is responding primarily in one mode, this total force will be proportional to the product of the displacement of the system and the modulus of the generalised complex stiffness of the mode. The r.m.s. value of the force will therefore be proportional to

$$q_{rms} \cdot K\sigma(1+\eta^2)^{1/2} .$$

Using equation IV.6, this becomes

$$\left[W_f(\omega_n)(\pi/2)\right]^{\frac{1}{2}} \cdot \left[M^{\frac{1}{4}} K^{-\frac{1}{4}} \mu^{\frac{1}{4}} \sigma^{-\frac{1}{4}} \eta^{\frac{1}{2}} (1+\eta^2)^{-\frac{1}{2}}\right]^{-1}. \qquad \ldots IV.9$$

Assuming once again that $\eta^2 \ll 1$, the efficiency criterion derived from this expression is evidently $\sigma^{-\frac{1}{4}}\eta^{\frac{1}{2}}$ per given weight of treatment (N.B. for convenience the resultant of the inertia and exciting forces will be referred to simply as the "inertia force" from now on).

It may be noted that the last term IV.9 may be written $(\eta/\omega_n)^{-\frac{1}{2}}$ (if $\eta^2 \ll 1$). Since this quantity must be as large as possible to minimize the random force, ω_n should obviously be as small as possible. It is therefore disadvantageous for a damping treatment to increase the natural frequency of the system, as far as these inertia forces are concerned, as this increase counteracts, in some measure, the benefits arising from the increased damping.

The criteria and related response expressions derived in this section are summarized in Table II.

IV.B. Sound Transmission through a single plate under random excitation.

Now suppose that the finite plate of section III.A is subjected on one side to random pressure fluctuations which give rise to a generalized force having a spectrum which is flat (or nearly flat), as before. The r.m.s. sound pressure transmitted by one of the plate modes of vibration to the far field on the other side will be proportional to the generalized r.m.s. acceleration of the plate mode. The effectiveness of a damping treatment in attenuating the resonant component of the transmitted pressure is therefore represented by the same expression as derived above for the r.m.s. acceleration. The corresponding efficiency criterion is again the value of $\sigma^{-\frac{1}{4}}\eta^{\frac{1}{2}}$ per given weight of treatment.

IV.C. Random Sound Transmission through an array of plates subjected to boundary layer pressure fluctuations.

As a further example of the different criteria which must be used for different systems and response quantities, reference will now be made to the work of Kraichnan (ref.4). This considers the acoustic radiation from an array of thin, independent, square plates on one side of which a moving airstream exerts convected boundary layer pressure fluctuations. After certain simplifying assumptions have been made, expressions are developed for the mean square radiated sound pressure (i.e. transmitted pressure) in terms of the mean square incident pressure and plate parameters. Many different modes of plate vibration contribute to the total radiation.

Under longitudinal dipole excitation, having a distributed convection velocity, the mean square radiation pressure $\langle p_r^2 \rangle$ is found to be of the form

$$\langle p_r^2 \rangle = \langle p^2 \rangle \cdot C_1 \cdot \left[(mB)^{1/2} (\mu\sigma)^{1/2} \eta \left(m\mu + \frac{C_2}{L^2} . B\sigma \right) \right]^{-1} . \qquad \text{...IV.10}$$

$\langle p^2 \rangle$ is the mean square incident pressure, and C_1 and C_2 are constants relating to the pressure distribution only. L is the length of the plate, $m\mu$ the mass per unit area, and $B\sigma$ the real part of the flexural rigidity of the plate. The expression is valid only for small values of η . The criterion for judging the efficiency of a damping treatment in attenuating the radiated r.m.s. sound pressure is now the value of $\left[\sigma^{1/2} \eta \left(\mu + \frac{C_2 B\sigma}{L^2 . m} \right) \right]^{1/2}$ per given value of μ . (The power $1/2$ is introduced to allow for the root mean square value. Clearly for the sake of indicating which is the best of a number of treatments, the square root need not in fact be taken).

When the incident pressure fluctuations derive from transverse dipole excitation and a sharp convection velocity, the mean square radiation pressure is of the form

$$\langle p_r^2 \rangle = \langle p^2 \rangle C_3 \left[m^{7/4} B^{1/4} \mu^{7/4} \sigma^{1/4} \eta \right]^{-1} , \qquad \text{...IV.11}$$

the corresponding efficiency criterion for a damping treatment in relation to the r.m.s. pressure being the value of $\sigma^{1/8}\eta^{1/2}$ per given weight.

One of the assumptions made in the derivation of the above expressions is that the damping (or loss) factor, η, is the same for each of the modes contributing to the radiation. When the damping derives from a visco-elastic damping treatment, this assumption is invalid for two reasons:

(a) The visco-elastic properties of the damping medium are considerably frequency dependent, and will therefore cause variations in η from mode to mode on account of the different natural frequencies involved.

(b) Constrained layer damping treatments give loss factors and flexural stiffnesses which depend on the wavelength of the flexural vibration. The treatments are designed to give optimum damping under certain conditions of geometry and wavelength. These conditions cannot be satisfied by all possible modes. The extension of Kraichnan's analysis to cover variations of damping factor and flexural stiffness with frequency is undoubtedly beset with great difficulties.

V. AN EXAMPLE OF THE USE OF THE CRITERIA.

In order to demonstrate the use of the criteria deduced in the preceding sections, a comparison will be made between three different treatments applied uniformly to one side of an aluminium plate (or strip) undergoing flexural vibrations. It will be shown that whereas one treatment may be superior in its effect when judged by one of the criteria, another treatment may be superior when judged by another criterion. Furthermore, one of the criteria may indicate that a given treatment is most effective when a certain optimum quantity is used, whereas another criterion may indicate that a different optimum quantity is required.

V.A. A Comparison of different treatments.

The treatments considered are two different grades of a commercial "unconstrained layer" type of treatment (referred to as treatment A and treatment B) and a commercial damping tape (treatment C). A and B consist of filled resins, different fillers being used in each material. The properties of the materials, as deduced from laboratory tests on simple treated specimens at a fixed temperature, are:

	Treatment A	Treatment B
Young's Modulus (real part)	860,000 lb/in.^{2}	1,080,000 lb/in.^{2}
Loss Factor	0·193	0·33
Specific Gravity	1·20	1·68

From these values curves have been obtained for the loss factor η and stiffness ratio, σ, of a uniform plate, covered uniformly with different quantities of the treatments (using expressions for η and σ first derived by Oberst, ref.5.). An aluminium plate has been considered, having a Young's Modulus of 10.8×10^6 lb/in.^{2} and a specific gravity of 2.84. σ and η have been plotted in figs. 2 and 3, the abscissa being the weight of the treatment as a fraction of the weight of the plate.

The damping tape consists of a thin aluminium foil adhering to which is a soft pressure sensitive damping material. The properties of the damping material alone are insufficient to permit an estimate being made of the loss factor and stiffness ratio of a treated plate, as the wavelength of the flexural vibration is now an important parameter in determining these quantities. In view of this, consideration is given only to the effect of the tape on one configuration, viz. a simply supported aluminium plate of width 4 inches and thickness 0.036 in., which is long compared with its width. Laboratory tests on a treated configuration of this type have yielded the following values for η and σ :

	η	σ
1 layer of tape	0.043	1.42
2 layers of tape	0.043	1.62

The weight of each layer of tape was 0.166 times that of the untreated plate. These values of η & σ have been superimposed upon the curves of η and σ for treatments A and B.

From these values of η and σ the values have been calculated of certain selected criteria from the preceding sections, and these are plotted in figs. 2,3, 4 and 5. The criterion relating to harmonic displacement amplitude ($\sigma\eta$) is shown in fig.3, together with the loss factor η, which is the criterion relating to harmonic sound pressure transmitted at resonance through a finite plate. Fig. 4 shows the criteria relating to random vibration amplitude ($\sigma^{3/4}\eta^{1/2}$), random inertia force ($\sigma^{-1/4}\eta^{1/2}$) and random sound pressure transmitted through an array of plates subjected to boundary layer turbulence.* Fig. 5 shows the criteria relating to the bending stress at the free surface of the plàte when vibrating under harmonic resonant or random conditions ($\sigma\eta\alpha^{-1}$ and $\sigma^{3/4}\eta^{1/2}\alpha^{-1}$ respectively).

It will be seen from fig. 2 that treatment A, which has a smaller Young's Modulus than treatment B, nevertheless provides a higher stiffness ratio σ than treatment B. This is due to the lower density of treatment A, which therefore has a greater thickness for a given weight of material and a correspondingly greater second moment of area about the neutral surface of the composite plate. This has an important bearing upon the criteria $\sigma\eta$ and $\sigma^{3/4}\eta^{1/2}$, which are shown in figs. 3 and 4. At low treatment weights the value of each of these criteria is higher for treatment B (with the superior material loss factor and Young's Modulus) than for A. At higher treatment weights this superiority is reversed due to treatment A providing the higher stiffness ratio. At the low treatment weights, where the stiffness ratio is little greater than unity, the two criteria approach the values of η and $\eta^{1/2}$ respectively, and the treatment providing the highest loss factor η is automatically superior.

Consider now the criterion $\sigma^{-1/4}\eta^{1/2}$ (fig.4), by which is judged the effect of a treatment on random inertia forces, or on the resonant component of random

* ($\sigma^{1/8}\eta^{1/2}$)

transmitted sound pressure. Since the stiffness ratio is raised to a negative power in this criterion, the treatment providing the lower stiffness ratio but the higher loss factor (treatment B) is inevitably superior over the whole weight range. It may be seen that equation III.10, giving the criterion relating to one form of coincidence transmission, contains $\eta\,\sigma^{-1/2}$ i.e. $(\sigma^{-1/4}\eta^{1/2})^2$. The criterion $\sigma^{-1/4}\eta^{1/2}$ may therefore be used in relation to both random inertia forces and this form of coincidence transmission.

The criterion $\sigma^{1/8}\eta^{1/2}$ (fig. 4) also shows treatment B to be superior up to, and beyond, a weight ratio of 10 on account of the higher loss factor and the very small power to which σ is raised. A maximum appears to occur at the weight ratio of about 10, but the curves will inevitably rise at higher (very impracticable) weight ratios on account of the positive power of σ and the asymptotic nature of η.

The criteria relating to the surface bending stresses in a plate are shown in Fig. 5. $\sigma\eta\alpha^{-1}$ relates to harmonic resonant conditions and $\sigma^{3/4}\eta^{1/2}\alpha^{-1}$ to random conditions. They follow a generally similar trend to the criteria relating to vibration displacement amplitudes. Over the lower weight range, treatment B is superior to A under both random and and harmonic conditions, but treatment A is superior under random conditions above a weight ratio of about 0.5. This superiority derives from the larger values of $\sigma^{3/4}\eta^{1/2}$ for treatment A above this weight, which implies smaller vibration amplitudes. The superiority of A over B does not, however, become more marked as the weight increases in the same manner as exhibited by $\sigma^{3/4}\eta^{1/2}$. This is due to the distance between the free plate surface and the composite plate neutral surface being greater for the lower density treatment A than for B. As the weight of treatment increases, this distance in the case of treatment A becomes increasingly greater than that of B. It is this latter effect that causes $\sigma\eta\alpha^{-1}$ for treatment A to be slightly inferior to that for B throughout the weight range, despite the superiority of $\sigma\eta$ for A above a weight ratio of about 0.75.

Comparing now the criteria corresponding to treatments B and C, it will be seen that apart from the value of σ, all the criteria for the two layer tape configuration are inferior to those of treatment B at the same weight. This is due to the low value of η for the two tape layers. The single layer of tape has a greater value of σ, and a very slightly greater value of η than has treatment B at the same weight. This automatically implies that the criteria involving positive powers of σ have superior values. However, the criterion involving the negative power of σ, ($\sigma^{-1/4}\eta^{1/2}$) is found to have a slightly inferior value to that of treatment B.

It should be emphasized at this stage that the above comparisons may not be used to formulate generalized statements regarding the relative merits of unconstrained layer treatments and constrained layer treatments. A particular example only has been chosen for the constrained layer configuration, and considerably different values of the criteria would be expected if the plate wavelength or tape thickness were to be changed.

V.B. A comparison of different amounts of the same treatment.

The criteria considered in the previous paragraph do not contain the mass ratio term μ. This has been omitted since it is not necessary to include it when equal weights of different treatments are being compared. It has been pointed out in the Introduction that a damping treatment may add considerably to the weight of thin light alloy structures (e.g. aeroplane structures). When considering the effect on the response of varying the quantity of a given treatment, the mass effect must therefore be included in the criterion used. Fig. 5 shows some of the criteria considered in the other figures, but with the appropriate mass ratio term included. $\mu^{7/8}\sigma^{-1/8}\eta^{1/2}$ is the criterion relating to the sound pressure transmitted due to boundary layer excitation. $\mu^{5/4}\sigma^{-1/4}\eta^{1/2}$ relates to r.m.s. (random) sound pressure transmitted through a single finite plate, and $\mu^{1/4}\sigma^{-1/4}\eta^{1/2}$ relates to r.m.s. (random) inertia forces. The first

two of these criteria increase steadily throughout the range of weights considered, but comparison with fig. 4 shows that over the upper part of the range the increase is due mostly (if not entirely) to the increasing mass. The third criterion shows that the treatments have a maximum effect upon random inertia forces at optimum treatment weights of about 0.6 and 0.75 of the weight of the plate for treatments A and B respectively. These compare with optimum weights of about 1.5 and 2.0 required to give maximum values to η for these treatments, i.e. giving minimum harmonic inertia forces. The existence of the maximum for η is a well known characteristic of unconstrained layer damping treatments.

The criterion relating to harmonic sound pressure transmitted through a finite plate ($\mu\eta$) will not exhibit the maximum shown by η alone for these treatments, but will rise steadily above a relative treatment weight of about 1, roughly in proportion to μ.

Equation II.4 shows that the harmonic displacement amplitude, and therefore the corresponding surface bending stress are independent of μ; the relevant curves of figs. 3 and 5 may therefore be considered in the present discussion. Each of these curves is monotonically increasing, implying that increasing the amount of the treatment will always provide a further reduction in the amplitude of resonant vibration and stress. The r.m.s. (random) displacement and stress <u>are</u> dependent upon the mass ratio μ, the corresponding criteria for the damping treatment being $\mu^{1/4}\sigma^{3/4}\eta^{1/2}$ and $\mu^{1/4}\sigma^{3/4}\eta^{1/2}\alpha^{-1}$. Since μ is raised to a positive power, these expressions will still increase monoticallly with increasing weight of treatment.

So far, the comparisons have been made between equal <u>weights</u> of different treatments. This is a valid basis for the aircraft manufacturer who is more concerned with the weight of the treatment that with the cost. When the cost is of greatest importance, it is obvious that the comparison should be made as follows:

Suppose treatment A cost X times that of B (for the same weight). The

criterion for treatment B at relative weight w should be compared with the criterion for treatment A at weight w/X. The criterion must include the appropriate term in μ. (i.e. $\mu^{\frac{1}{4}}\sigma^{-\frac{1}{4}}\eta^{\frac{1}{2}}$, $\mu\eta$ etc.)

VI CONCLUSIONS

In this paper, expressions have been derived for the response of simple systems which have been treated with damping materials and which are subjected to harmonic or randomly varying forces. These expressions include the effect of the mass, stiffness and damping of the treatments. From each of the different expressions for the response quantities considered, a criterion has been found by which to compare the effectiveness of different damping treatments and the effect of different amounts of the same treatment. The importance of including the effect of the mass and stiffness of the treatment has been emphasized. On the basis of these criteria, it has been show that for some of the response quantities (e.g. vibration displacement amplitude) it is an advantage for the treatment to increase the stiffness of the system, whereas for others (e.g. inertia forces under random excitation or a certain form of acoustic transmission by the coincidence effect) it is a disadvantage. In no case, however, is it a disadvantage to increase the mass of the system.

In the example to illustrate the use of the criteria, it has been shown that with relatively small weights of damping treatment, the treatment giving the greatest loss factor to the whole system is superior as judged by each of the criteria. This follows from the fact that at small relative weights the stiffness and mass increases are negligible for the particular damping configurations considered. The loss factor is then the only parameter which has been changed appreciably by the addition of the treatment. At greater relative weights, it has been shown that one treatment having a lower density, a lower stiffness and a lower material loss factor than another can, nevertheless, be more efficient (on an equal weight basis) in attenuating vibration displacement amplitudes and plate surface bending stresses.

The criteria relating to harmonic and random inertia forces show that there are optimum quantities of treatment to give the greatest effects, but the optimum quantities differ for the harmonic and random conditions. If the amount actually used is mid-way between these two values, the reduction in effectiveness below the maximum realizeable is very slight. The other criteria considered are all monotically increasing with increase of treatment weight, implying that increasing the amount of treatment used will always further reduce the response.

The implications of these results are that when damping treatments are being considered for use on light aluminium structures, their effectiveness cannot be sufficiently defined by stating only the loss factor obtainable from a given amount of the treatment. The factor by which the stiffness of the structure is increased must also be given. This implies that the results of the standard Geiger test, whereby the time rate of decay of a treated steel plate is given as the measure of the effectiveness of the treatment, is also insufficient. This time rate of decay is (in effect) but an alternative form of presenting the value of the loss factor (ref. 6).

The fact that a poorer quality treatment has been shown to have a superior effect, in some instances, than one of higher quality, suggests that the optimum design or compounding of a treatment will be different depending on the particular vibration response quantity it is required to attenuate. It may be that damping treatments can be developed further along these lines, a different treatment being designed and recommended for different applications.

The inclusion of the mass and stiffness effects into the criteria for assessing damping treatments will be more than ever important when "space" damping techniques are being considered, since these techniques can be expected to give a very large stiffness increase for relatively small weights of treatment.

REFERENCES

1. D.J. Mead, Aeronautical Engineering, Vol.32, (April 1960)
2. J.H. Foxwell and R.E. Franklin, Aeronautical Research Council Report No. ARC.19,743 (1957)
3. J.H. Foxwell, and R.E. Franklin, Aeronautical Quarterly, 10 (1959)
4. R.H. Kraichnan, Jour. Acoustical Soc. America, 29, 65 - 80 (1957)
5. H. Oberst, Acustica 2, Akus. Beih. No.4, 181-194 (1952)
6. E. Kerwin, Jour. Acoustical Soc. America, 31, 952-962, (1959)

Response Quantity	Frequency For Maximum Response	Maximum Value Of Response	Value Of Response At ω_n	Treatment Criterion (small η)	Reference
Vibration Displacement	ω_n	$P/K\sigma\eta$	$P/K\sigma\eta$	$\sigma\eta$	II.4
Plate Bending Stress	ω_n	$P\alpha/K\sigma\eta$	$P\alpha/K\sigma\eta$	$\sigma\eta\alpha^{-1}$	§ II.B
Velocity	$\omega_n(1+\eta^2)^{\frac{1}{4}}$	$P/K^{\frac{1}{2}}M^{\frac{1}{2}}\sigma^{\frac{1}{2}}\mu^{\frac{1}{2}}[2(1+\eta^2)^{\frac{1}{2}}-2]^{\frac{1}{2}}$	$P/K^{\frac{1}{2}}M^{\frac{1}{2}}\sigma^{\frac{1}{2}}\mu^{\frac{1}{2}}\eta$	$\sigma^{\frac{1}{2}}\mu^{\frac{1}{2}}\eta$	II.5,6
Acceleration and Sound Pressure Transmitted by Finite Plate	$\omega_n(1+\eta^2)^{\frac{1}{2}}$	$P/M\eta(1+\eta^2)^{-\frac{1}{2}}$	$P/M\mu\eta$	$\mu\eta$	II.7,8
Inertia Force	$\omega_n(1+\eta^2)^{\frac{1}{2}}$	$P/\eta(1+\eta^2)^{-\frac{1}{2}}$	P/η	η	II.9

Table 1 Summary of Expressions for the Maximum Values of Harmonic Response and the Corresponding Treatment Criteria

Response Quantity	r.m.s. Response	Treatment Criterion	Equation No.
Displacement	$\left[W_f(\omega_n)(\pi/2)\right]^{\frac{1}{2}}.\left[M^{\frac{1}{4}}K^{\frac{3}{4}}\mu^{\frac{1}{4}}\sigma^{\frac{3}{4}}\eta^{\frac{1}{2}}\right]^{-1}$	$\mu^{\frac{1}{4}}\sigma^{\frac{3}{4}}\eta^{\frac{1}{2}}$	IV.6
Velocity	$\left[W_f(\omega_n)(\pi/2)\right]^{\frac{1}{2}}.\left[M^{\frac{3}{4}}K^{\frac{1}{4}}\mu^{\frac{3}{4}}\sigma^{\frac{1}{4}}\eta^{\frac{1}{2}}\right]^{-1}$	$\mu^{\frac{3}{4}}\sigma^{\frac{1}{4}}\eta^{\frac{1}{2}}$	IV.7
Acceleration ($\propto$ Sound Pressure through plate, IV.B)	$\left\{ W_f(\omega_n)(\pi/2)(1-\eta^2)\left[M^{\frac{5}{2}}K^{-\frac{1}{2}}\mu^{\frac{5}{2}}\sigma^{-\frac{1}{2}}\eta\right]^{-1} + \langle F^2(t)\rangle/M^2\mu^2 \right\}^{\frac{1}{2}}$	$\mu^{\frac{5}{4}}\sigma^{-\frac{1}{4}}\eta^{\frac{1}{2}}$	IV.8
Inertia force	$\mathrm{Const.}\left[W_f(\omega_n)(\pi/2)\right]^{\frac{1}{2}}.\left[M^{\frac{1}{4}}K^{-\frac{1}{4}}\mu^{\frac{1}{4}}\sigma^{-\frac{1}{4}}\eta^{\frac{1}{2}}(1+\eta^2)^{-\frac{1}{2}}\right]^{-1}$	$\mu^{\frac{1}{4}}\sigma^{-\frac{1}{4}}\eta^{\frac{1}{2}}$	IV.9
Boundary layer Sound Pressure Transmitted	$\mathrm{Const.}\langle p^2\rangle.\left[m^{\frac{7}{8}}B^{\frac{1}{8}}\mu^{\frac{7}{8}}\sigma^{\frac{1}{8}}\eta^{\frac{1}{2}}\right]^{-1}$	$\mu^{\frac{7}{8}}\sigma^{\frac{1}{8}}\eta^{\frac{1}{2}}$	IV.11

Table II Summary of Expressions for r.m.s. Random Response and the Corresponding Treatment Criteria.

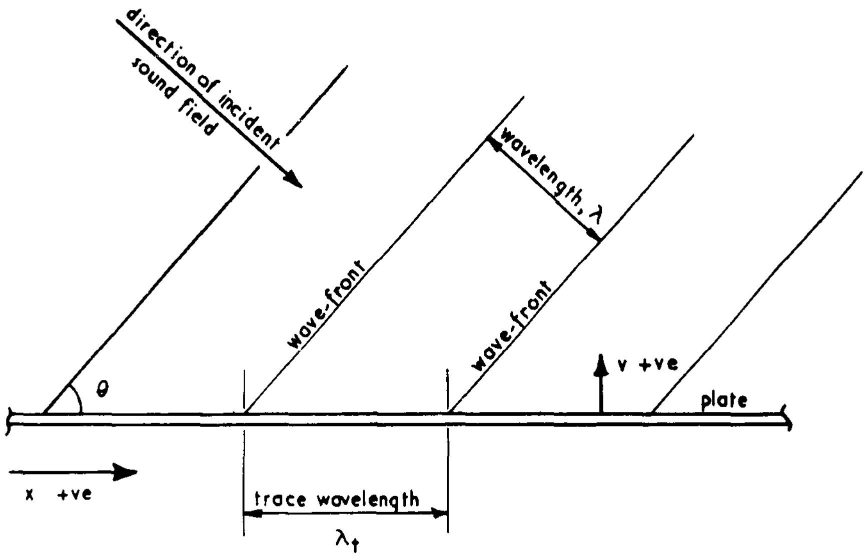

FIG. 1. DIAGRAM ILLUSTRATING SYMBOLS USED IN PARA. III.B

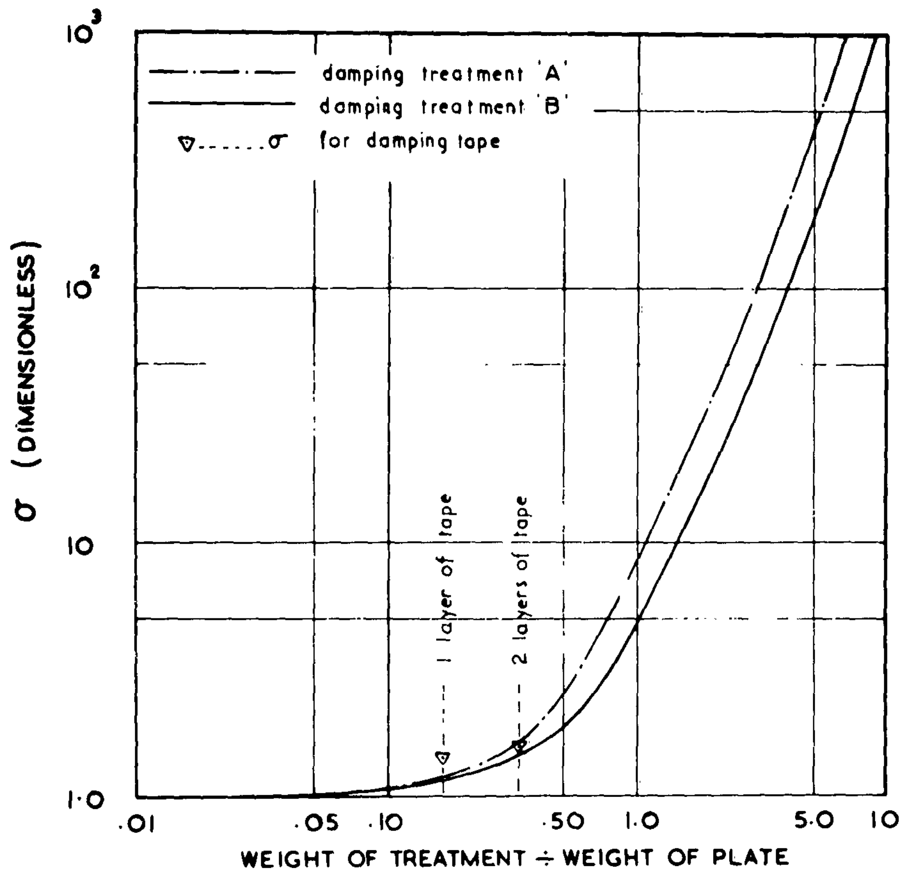

FIG. 2 THE STIFFNESS RATIO σ AS A FUNCTION OF RELATIVE WEIGHT OF TREATMENT.

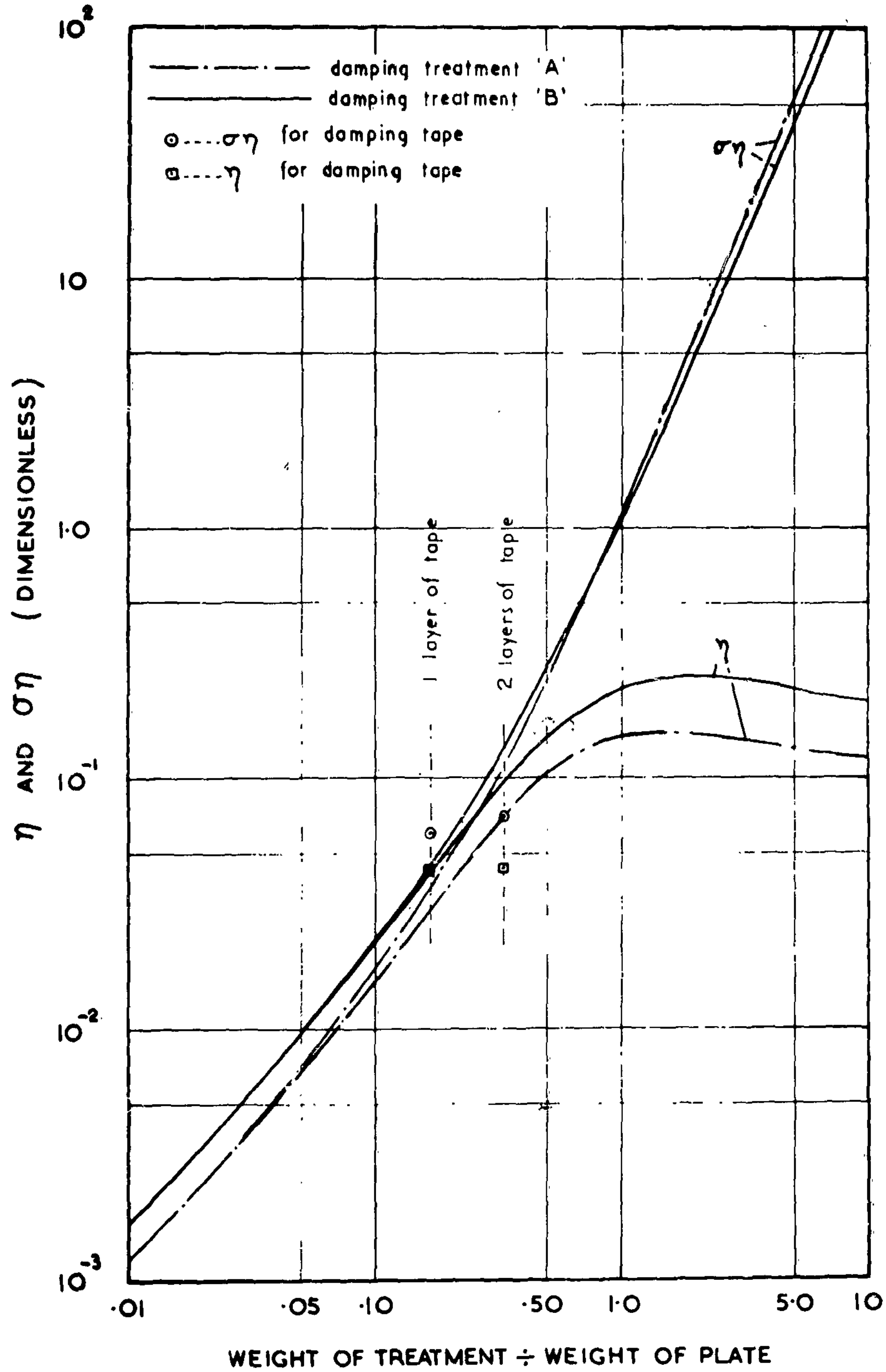

FIG. 3 THE LOSS FACTOR η AND THE CRITERION $\sigma\eta$ FOR THE TREATED PLATE AS A FUNCTION OF RELATIVE WEIGHT OF TREATMENT.

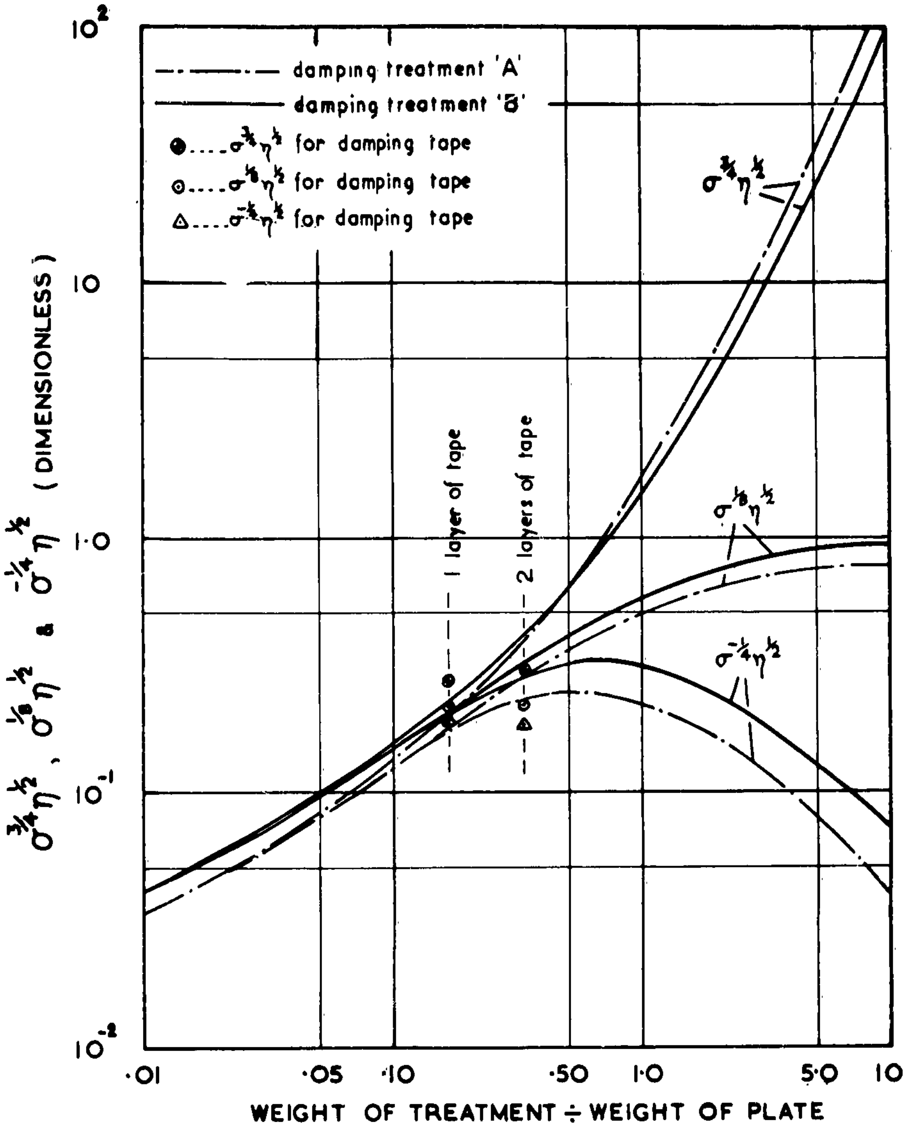

FIG. 4 CRITERIA RELATING TO RANDOM DISPLACEMENT, SOUND PRESSURE TRANSMITTED AND INERTIA FORCE AS A FUNCTION OF RELATIVE WEIGHT OF TREATMENT. (MASS EFFECT NOT INCLUDED)

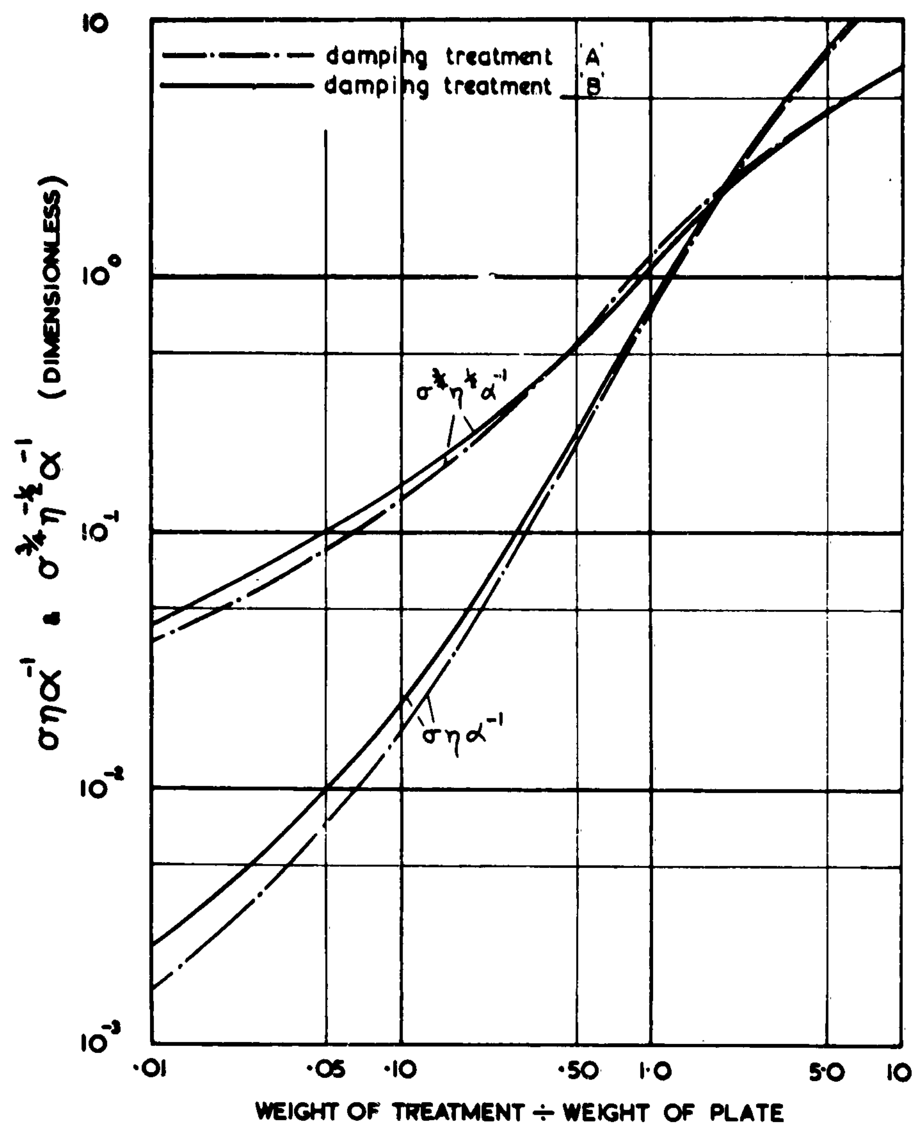

FIG. 5 CRITERIA RELATING TO PLATE SURFACE BENDING STRESS AS A FUNCTION OF RELATIVE WEIGHT OF TREATMENT. (MASS EFFECT NOT INCLUDED)

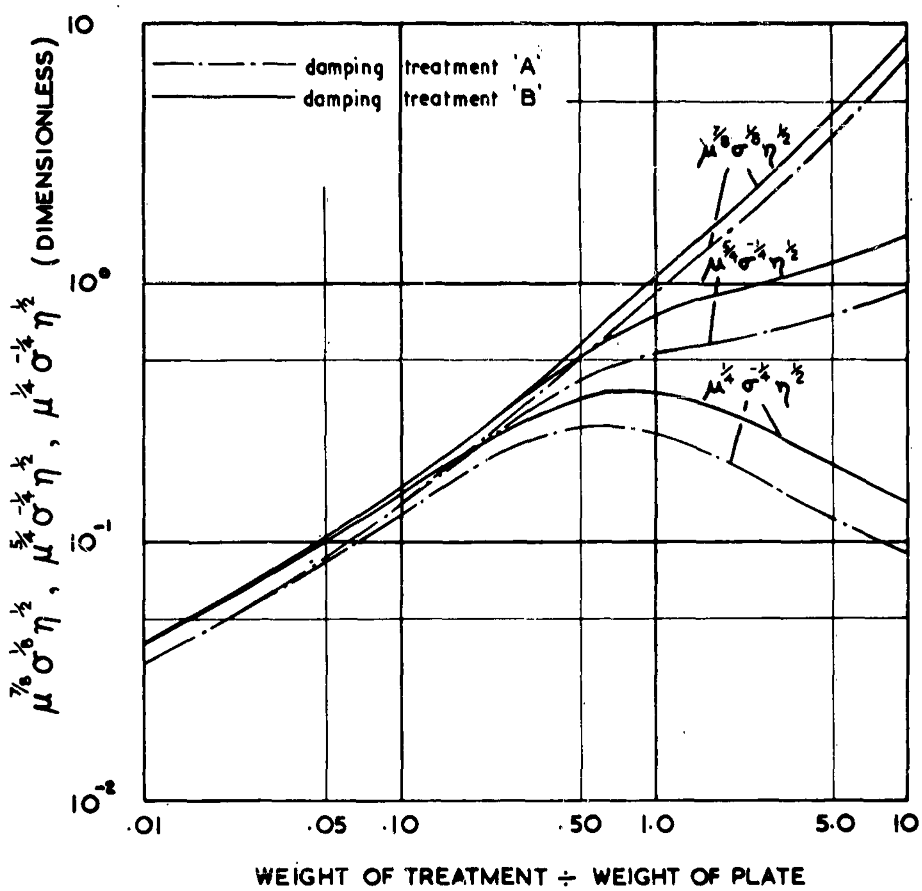

FIG. 6 CRITERIA RELATING TO RANDOM TRANSMITTED SOUND PRESSURE, RANDOM ACCELERATION AND RANDOM INERTIA FORCE AS A FUNCTION OF RELATIVE WEIGHT OF TREATMENT. MASS EFFECT INCLUDED.

UNCLASSIFIED

UNIVERSITY OF SOUTHAMPTON, Hampshire, England, CRITERIA FOR COMPARING THE EFFECTIVENESS OF DAMPING TREATMENTS, by D. J. Mead, January 1961. 36p. incl. figs. and tables. (Project 7351; Task 73521) (WADD TR 61-25) (Contract AF 61(052)-332)

Unclassified report

Criteria have been deduced indicating the effectiveness of a damping treatment in attenuating bending stresses, accelerations, inertia forces and sound pressure transmitted associated with simple vibrating plates under harmonic and random excitation. Coincidence sound transmission through plates is also considered. The importance is emphasized of including in these criteria

(over)

UNCLASSIFIED

UNCLASSIFIED

the factors by which the damping treatment increases the mass and stiffness of the system. Three different commercial treatments have been compared using some of the criteria and it has been shown that the treatment providing the highest loss factor increment to the system is not necessarily the most effective in reducing all the response quantities.

UNCLASSIFIED

UNCLASSIFIED

UNIVERSITY OF SOUTHAMPTON, Hampshire, England, CRITERIA FOR COMPARING THE EFFECTIVENESS OF DAMPING TREATMENTS, by D. J. Mead, January 1961. 36p. incl. figs. and tables. (Project 7351; Task 73521) (WADD TR 61-25) (Contract AF 61(052)-332)

Unclassified report

Criteria have been deduced indicating the effectiveness of a damping treatment in attenuating bending stresses, accelerations, inertia forces and sound pressure transmitted associated with simple vibrating plates under harmonic and random excitation. Coincidence sound transmission through plates is also considered. The importance is emphasized of including in these criteria

(over)

UNCLASSIFIED

UNCLASSIFIED

the factors by which the damping treatment increases the mass and stiffness of the system. Three different commercial treatments have been compared using some of the criteria and it has been shown that the treatment providing the highest loss factor increment to the system is not necessarily the most effective in reducing all the response quantities.

UNCLASSIFIED